Physics 211C: Solid State Physics

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Lecture 11

Topic: Heavy Fermi Liquids

specifically large eff. mass & small Z

next up: Quantum magnetism (LSM theorem)

HFL MFT :-

$$S = f^{+} \frac{\sigma}{2} f$$

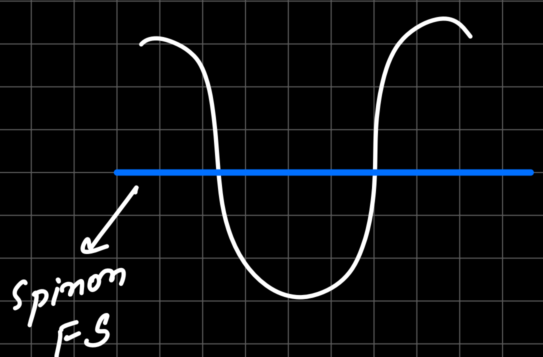
$$\mathcal{H}_{MF} = \begin{bmatrix} c^{+}_{k\sigma} & f^{+}_{k\sigma} \end{bmatrix} \begin{bmatrix} \varepsilon_k & -V \\ -V & \mu_f \end{bmatrix} \begin{bmatrix} c_{k\sigma} \\ f_{k\sigma} \end{bmatrix}$$

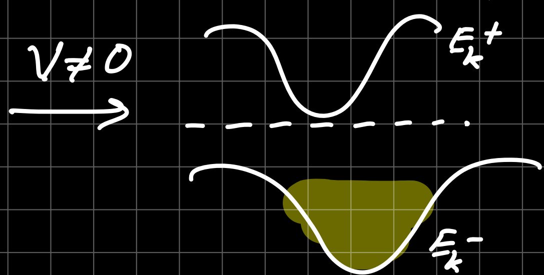
ensures $\langle f^{+} f \rangle = 1$

$V = \langle c^{+} f \rangle \equiv$ Higgs condensate

↳ gaps out the gauge field

↳ Low en. gapless modes like photon don't exist, "higgsing"





$n_{spin} = 1$

$n_c < 1$

Vol. of fermi sea $= n_c + n_{spin}$

$$V = \frac{J_K}{2} \langle c^\dagger f + h.c. \rangle \Rightarrow V \simeq E_F \, e^{-\frac{1}{N(E_F) J_K}}$$

Heaviness of bands (eff. mass of q.p.) :-

$$E_k^- \simeq \mu_f - \frac{V^2}{\varepsilon_k - \mu_f}$$

$$E_F \approx \mu_f - \frac{V^2}{\varepsilon(k_F) - \mu_f}$$

$$N(E_F) \simeq \int d\varepsilon \, \rho_c(\varepsilon) \, \delta\left(E_F - \mu_f + \frac{V^2}{\varepsilon - \mu_f}\right)$$

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}$$

$$\varepsilon - \mu_f = (\mu_f - E_F)^{-1} V^2 \Rightarrow \varepsilon = \mu_f +$$

$\vdots$

$$N(E_F) \simeq \int d\varepsilon \, \rho_c(\varepsilon) \left[\frac{V^2}{\varepsilon - \mu_f} - \frac{V^2}{\varepsilon(k_F)} - \mu_f \right]$$

$$= \frac{\rho_c(\varepsilon(k_F))}{\left| \frac{\partial}{\partial \varepsilon} \left(\frac{V^2}{\varepsilon - \mu_f} \right) \right|_{\varepsilon(k_F)}}$$

$$= \underbrace{\rho_c(\varepsilon(k_F))}_{\text{dos w/o hybridisat}^n} \underbrace{\frac{(\varepsilon(k_F) - \mu_f)^2}{V^2}}_{\text{enhancement}} \sim \frac{t^2}{V^2}$$

$$\frac{m^*}{m} \sim \exp\left(\frac{2}{N(E_F) J_K} \right)$$

Quasiparticle residue Z:-

M1: $Z \sim \dfrac{m}{m^*} \sim e^{-\frac{2}{N(E_F) J_K}}$ (from FLT arguments)

M2:

$$G(k, i\omega)$$
$$\uparrow$$
$$G = \langle c(k,\tau)\, c^\dagger(k,\tau) \rangle$$

$$\mathcal{H}_{MF} = [c^\dagger_{k\sigma} \;\; f^\dagger_{k\sigma}]\, h_k \begin{bmatrix} c \\ f \end{bmatrix} \qquad U_k^\dagger h U_k = \begin{bmatrix} E_k^+ & 0 \\ 0 & E_k^- \end{bmatrix}$$

$$U_k = \begin{bmatrix} \cos\theta_k & \sin\theta_k \\ \sin\theta_k & -\cos\theta_k \end{bmatrix} \xrightarrow[\text{that}]{\text{show}} \cos^2\theta_k = 1 - \frac{V^2}{t^2}$$

$$\therefore [c^\dagger \; f^\dagger]\, h \begin{bmatrix} c \\ f \end{bmatrix} = \psi^\dagger h \psi = \underbrace{(\psi^\dagger U)}_{\gamma^\dagger} \underbrace{(U^\dagger h U)}_{D} \underbrace{(U^\dagger \psi)}_{\gamma}$$

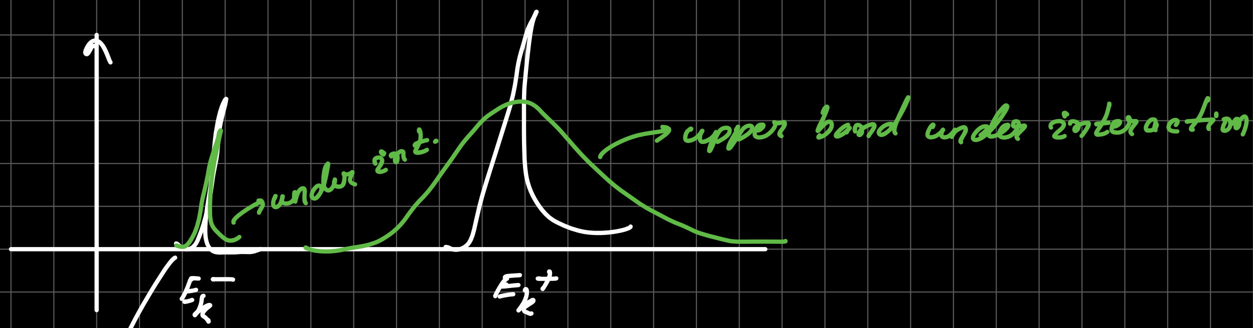
$$\gamma = \begin{pmatrix} \gamma_+ \\ \gamma_- \end{pmatrix}$$

$$\mathcal{H} = \sum_{k\sigma} E_k^+ \, \gamma^\dagger_{k,+,\sigma} \gamma_{k,+,\sigma} + \sum_{k\sigma} E_k^- \, \gamma^\dagger_{k,-,\sigma} \gamma_{k,-,\sigma}$$

$$\langle c_k(k,\tau)\, c^\dagger(k,\tau=0) \rangle$$
$$= \cos^2\theta_k \langle \gamma_+(k,\tau)\, \gamma_+^\dagger(k,\tau) \rangle + \sin^2\theta_k \langle \gamma_-(k,\tau)\, \gamma_-^\dagger(k,\tau=0) \rangle$$
$$= \cos^2\theta_k \, e^{-E_k^+ \tau} + \sin^2\theta_k \, e^{-E_k^- \tau}$$
$$- \frac{\cos^2\theta_k}{i\omega - E_k^+} + \frac{\sin^2\theta_k}{i\omega - E_k^-}, \qquad \sin^2\theta_k \rightarrow Z$$

$$Z = \sin^2\theta_k \simeq \frac{V^2}{t^2}$$ } why arpes expts are tough?

$$A(k,\omega) = \mathrm{Im}\, G$$



— remains same i.e. peak is protected due to FLT arguments.

DISCUSSION ON PHASES of matter that aren't FLT

$$(\partial_\tau - \varepsilon_k)\,\bar{c}_k c_k + (\bar{f}_r f_{r'} e^{i a_{rr'}} + h.c.)$$

$$+ (\bar{f} c\, f \bar{c} + h.c.) + (\nabla \times a)^2$$

a — int. gauge field; A — EM

so far we considered

	a	A
f	1	0
c	0	1

HFL: $\langle \bar{f} c \rangle \neq 0 \to$ gives higgs effect & gaps out $(\nabla \times a)^2$

$\langle \bar{f} c \rangle = 0$?

Low energy:

$$(\partial_\tau - \varepsilon_k)\,\bar{c}_k c_k$$
$$+$$
$$(\bar{f}_r f_{r'}\, e^{i a_{rr'}} + h.c.)$$
$$-\mu_f (\bar{f} f)$$
$$+ (\nabla \times a)^2$$

decoupled i.e. no higgsing

what happens? ① FS is low $\to$ Quantum oscillations are less.

② $(\nabla \times a)^2 \to$ emergent photons

$\bar{f}_r f_{r'} \to$ emergent spinons

James Analytis et al (recent)

Kitaev's honeycomb model $\to$ exactly soluble model with emergent gauge fields & spinons

CeCoIn5

NFL (Luttinger theorem fails)

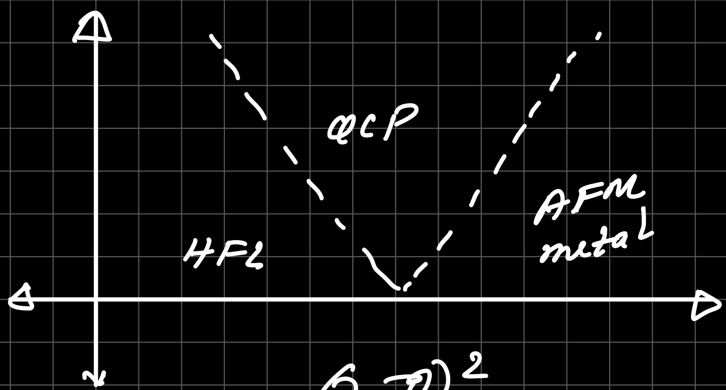
HFL ——×—— FL* ——×——→

senthil, Vojta, Sachdev

FS vol. doesn't match

"CePdAl"

FL* → spins decouple from CB, form Top. ord. phase



"Hertz-Millis-Moriya" theory

$(\partial_\mu \vec{N})^2$

↳ O(3) O.P.

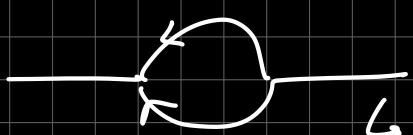
$$\int |N|^2 (|\omega| + q^2) + |N|^4 - - - - + - - - -$$

dissipative coupling

↳ integrating out

$N(r) = N(x) e^{i \vec{Q} \cdot \vec{r}}$

$N c^\dagger \sigma c (-1)^{x+y}$



↳ Lindhard susceptibility $q \sim Q$

Quantum Magnetism

- gapless phases
  - Goldstone modes
  - Gapless fractionalized excitations
    - ⇓ Heisenberg chain in 1d
- Gapped
  - Topo ordered phases
  - SPT

$+|J| \sum_r S_r \cdot S_{r+1}$: 1d Heisenberg

⇓ has gapless fractionalized excitations

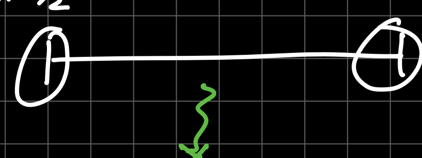
⇙ Low energy theory is some c.f.t

QSL: no local O.P., can't be obtained from a product state via a low depth unitary circuit.

LSM theorem

- spin 1/2 — gapless
- vs
- spin 1 — gapped (SPT phase)

Spin ½ objects at ends of a spin 1 system

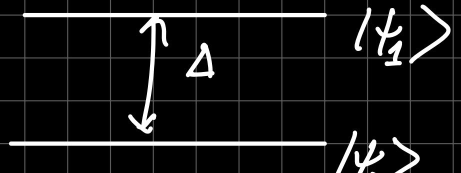


SPT phase

We want to talk about spin-s chains & investigate low lying spectrum.

Spin-s Heisenberg AFM chain $d = 1$

$O(N)$ NLSM $d=1$ @ $T=0$ is gapped. Exp. decaying correlation function



$\Delta \neq 0$ as $L \to \infty$

\# rotor model in 1d: $\sum \frac{n_i^2}{2I} + \sum \cos(\theta_i - \theta_{i+1})$ $\quad O(2)$ model

$[n, \theta] = i$

$\Leftarrow$ $O(N)$ model in 1d

can show

$$e^{-\frac{1}{g^2}\int (\partial_\mu n)^2 \, dx \, d\tau} \qquad \frac{+d(g^2)}{dl} = +g^2 (N-2)$$

Result:- When spin s is $\frac{1}{2}$ odd integer, one can construct a low lying state with energy E_1 s.t. $E_1 - E_0 \leq \frac{1}{L}$

Arg.:- $\quad T_x |\psi_0\rangle = e^{ik_0} |\psi_0\rangle$

$$|\psi_1\rangle = U|\psi_0\rangle = e^{\frac{2\pi i}{L} \sum_{x=1}^{L} x S^z(x)} |\psi_0\rangle$$

PBC ($S_{L+1} = S_1$)

① $T_x |\psi_1\rangle = e^{i(k_0 + \pi)} |\psi_1\rangle$

② $\langle \psi_1 | H | \psi_1 \rangle = E_1 \qquad E_1 - E_0 < \frac{1}{L}$

for spin $\frac{1}{2}$

$$T_x |\psi_1\rangle = T_x U |\psi_0\rangle = (T_x U T_x^\dagger) \, T_x |\psi_0\rangle$$
$$= (\; \cdot \cdot \cdot \;) \, e^{ik_0} |\psi_0\rangle$$

$$T_x S^z_x T_x^\dagger = S^z_{x+1} \quad \therefore \quad T_x \mathcal{U} T_x^\dagger = e^{\frac{2\pi i}{L}\sum_{x=1}^{L} x \, S^z(x+1)}$$

$$= \exp\left(\frac{2\pi i}{L}\, S^z(2) + 2S^z(3) + 3S^z(4) \cdots\right)$$

$$= \mathcal{U}\, e^{-\frac{2\pi i}{L}\underbrace{\sum_{x=1}^{L} S^z(x)}} \, e^{2\pi i S^z(1)}$$

$$\Downarrow$$

$\sum_{x=1} S^z(x) = S^z_{total} = 0$ in true ground state (Auerbach #5.1)

$$e^{i2\pi S^z} = +1 \text{ for int. spin.}$$

$$= -1 \text{ for } \tfrac{1}{2} \text{ int.}$$

$$S^z = -S, -S+1 \cdots, S$$

$$= -1 \text{ for } \tfrac{1}{2} \text{ odd int.}$$

$$\mathcal{H} = \sum_x S^z(x) S^z(x+1) + \frac{1}{2}\left(S^+(x) S^-(x+1) + h.c.\right)$$

$$S^+ = S_x + i S_y$$

$$\langle \Psi_0 | \mathcal{U}^\dagger \mathcal{H} \mathcal{U} | \Psi_0 \rangle = E_1$$

$$e^{-i\alpha S^z} S^{+/-} e^{i\alpha S^z} \qquad [S^z, S^+] = S^+$$

$$= e^{\mp i\alpha} S^{+/-}$$

$$\mathcal{U}^\dagger S^+(x)\, \mathcal{U} = e^{-i\frac{2\pi x}{L}} S^+(x)$$

$$\mathcal{U}^\dagger S^-(x+1)\, \mathcal{U} = e^{\frac{2\pi i}{L}(x+1)} S^-(x+1)$$

$$E_1 = \langle \Psi_0 | \sum_x S^z_x S^z_{x+1} | \Psi_0 \rangle + \frac{1}{2} e^{\frac{2\pi i}{L}} \langle \Psi_0 | \sum_x S^+_x S^-_{x+1} | \Psi_0 \rangle$$

$$+ \frac{1}{2} e^{-\frac{2\pi i}{L}} \langle \Psi_0 | \sum_x S^-_x S^+_{x+1} | \Psi_0 \rangle$$

$$\Rightarrow E_1 = E_0 + \frac{1}{2}\left(\cos\frac{2\pi}{L} - 1\right)\langle \psi_0 | \sum_x \underbrace{S_x^+ S^-_{(x+1)} + h.c.}_{= S^x S^x + S^Y S^Y} | \psi_0\rangle$$

$$+ \frac{i}{2}\sin\left(\frac{2\pi}{L}\right) \underbrace{\langle \psi_0 | \sum_x \underbrace{S_x^+ S_{x+1}^- - h.c.}_{= S^x S^Y_{x+1} - S^Y_x S^X_{x+1}} | \psi_0\rangle}_{= 0}$$

because of symmetry

$S^X \to S^Y$

define antiunitary

$S^X \to S^Y$

$S^Y \to S^X$

$S^Z \to S^Z$

$i \to -i$

$$\simeq E_0 + \frac{1}{L^2} \cdot \overbrace{S^2}^{\text{bounded.}} \cdot L \sim E_0 + \frac{1}{L}$$

in d-dim. $\frac{L^D}{L^2} \sim L^{D-2}$ (in $D \geq 2$, we can't say anything)

⇓

thats where oshikawa's arg. comes in.